Nonlinear and time-dependent equivalent-barotropic flows with topography

Luis Zavala Sansón[1]

[1]Centro de Investigación Científica y de Educación Superior de Ensenada, México

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Abstract

Some oceanic and atmospheric flows may be modelled as equivalent-barotropic systems, in which the horizontal fluid velocity varies in magnitude at different vertical levels while keeping the same direction. The governing equations at a specific level are identical to those of a homogeneous flow over an equivalent depth, determined by a pre-defined vertical structure. Most oceanic studies using the equivalent-barotropic approach are focused on steady, linear formulations. In this work, the nonlinear, time-dependent model with variable topography is examined. To include nonlinear terms, we assume suitable approximations and evaluate the associated error in the dynamical vorticity equation. The model is solved numerically to investigate the equivalent-barotropic dynamics in comparison with a purely barotropic flow. We consider two problems in which the behaviour of homogeneous flows has been well-established either experimentally or analytically in past studies. First, the nonlinear evolution of cyclonic vortices around a topographic seamount is examined. It is found that the vortex drift induced by the mountain is modified according to the vertical structure of the flow. When the vertical structure is abrupt, the model effectively isolates the surface flow from both inviscid and viscous topographic effects (due to the shape of the solid bottom and Ekman friction, respectively). Second, the wind-driven flow in a closed basin with variable topography is studied (for a flat bottom this is the so-called Stommel problem). For a zonally uniform, negative wind-stress curl in the homogeneous case, a large-scale, anticyclonic gyre is formed and displaced southward due to topographic effects at the western slope of the basin. The flow reaches a steady state due to the balance between topographic, β, wind-stress and bottom friction effects. However, in the equivalent-barotropic simulations with an abrupt vertical structure, such an equilibrium cannot be reached because the forcing effects at the surface are enhanced, while bottom friction effects are reduced. As a result, the unsteady flow is decomposed as a set of planetary waves.

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L. Zavala Sansón (lzavala@cicese.mx)
Department of Physical Oceanography, CICESE, México (www.cicese.mx/∼lzavala)

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Abstract

A nonlinear, time-dependent model for an equivalent-barotropic flow is examined. The model is solved numerically to investigate the equivalent-barotropic dynamics of experimental and observational examples in comparison with a purely barotropic flow.

Some antecedents

The motion of an equivalent-barotropic flow varies in magnitude at different vertical levels while keeping the same direction. The governing equations at a specific level are identical to those of a homogeneous flow over an equivalent depth, determined by a pre-defined vertical structure. The idea was proposed by Charney (1949) [1] for modelling a barotropic atmosphere. More recently, steady, linear formulations have been used to study oceanic flows, especially the Antarctic Circumpolar Current [2,3].

1 Physical model

Quasi-2D, shallow-water flow on a β-plane with fluid depth (h, y), velocity $(\boldsymbol{v}, w)$ and vertical structure $P(z)$ [4]:

$$\begin{aligned} \boldsymbol{v}(x,y,z,t) &= P(z)\boldsymbol{v}_s(x,y,t), \\ p(x,y,z,t) &= P(z)p_s(x,y,t), \end{aligned} \quad (1)$$

where subindex s indicates surface values. Vertical scales:

$$F(x,y) = \int_{-h}^{0} P(z)dz, \qquad G(x,y) = \int_{-h}^{0} P^2(z)dz \quad (2)$$

Horizontal velocity components in terms of transport function:

$$u_s = \frac{1}{F}\frac{\partial \psi}{\partial y}, \qquad v_s = -\frac{1}{F}\frac{\partial \psi}{\partial x} \quad (3)$$

Vertical velocity:

$$w(x,y,z,t) = \left(\int_z^0 P(z')dz'\right)\nabla\cdot\boldsymbol{v}_s \quad (4)$$

In this study $P(z)$ is exponential with reference depth z_0:

$$P(z) = e^{z/z_0}, \quad (5)$$

Vorticity equation including wind-stress forcing $\boldsymbol{\tau}$ and bottom friction with coefficient R:

$$\frac{\partial\omega_s}{\partial t} + J\left(\frac{\omega_s + f}{F}, \psi\right) = \nu\nabla^2\omega_s + \frac{1}{\rho_0}\left(\nabla\times\frac{\boldsymbol{\tau}_s}{F}\right) - R\left(\frac{1}{F} - \frac{1}{z_0}\right)\omega_s$$

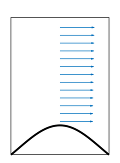
with $\omega_s = -\dfrac{1}{F}\nabla^2\psi + \dfrac{1}{F^2}\nabla F\cdot\nabla\psi.$

Error in the nonlinear terms associated with separation (1):

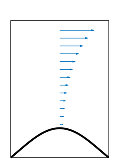
$$\max\left\{1 - \frac{G}{F}\right\}. \quad (6)$$

Limits:

$z_0 >> h \Rightarrow F \approx h(x,y)$ $\rightarrow$ homogeneous



$z_0 << h \Rightarrow F \approx z_0$ $\rightarrow$ shallow layer



2 Simulations of vortices around seamounts

- Strongly nonlinear, cyclonic vortices, f-plane (rotation period $T = 30$ s), no external forcing.
- Comparison with laboratory experiments with barotropic vortices performed in the Coriolis platform (Grenoble, France) [5].
- Aim: to illustrate how the vortex drift is modified according to z_0 (*i.e.* the effect of the vertical structure).

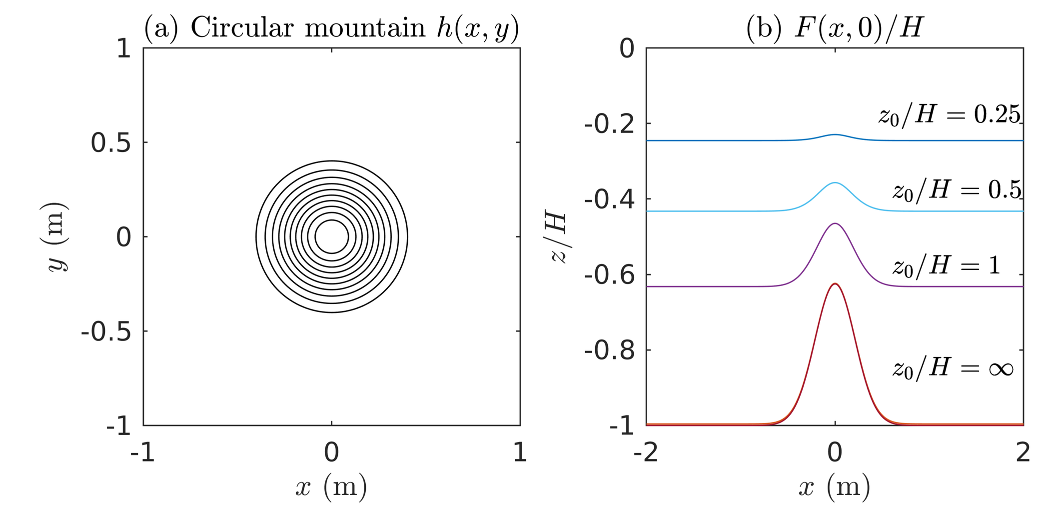


Figure 1: (a) Topography contours of a Gaussian mountain in a fluid with maximum depth H. (b) Equivalent depth profiles $F(x,0)/H$ calculated for different z_0.

$$\omega_s(x,y)P(z),\ \text{depth}\ h(x,y) \quad \equiv \quad \omega_s(x,y),\ \text{depth}\ F(x,y)$$

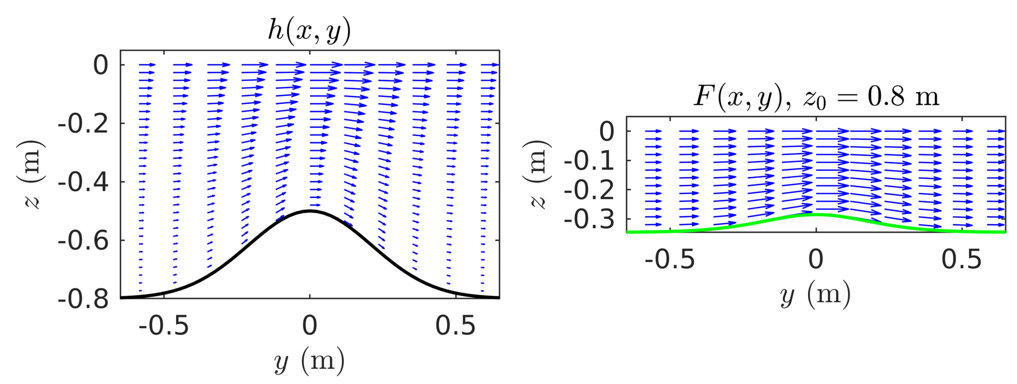


Figure 2: Vertical velocity profile for $z_0/H = 0.8$. Left: Flow with vertical structure $P(z)$ and depth $h(x,y)$. Right: Uniform flow with depth F(x,y). The surface flow in both cases is exactly the same.

The homogeneous vortex (panel a) drifts around the mountain in a clockwise direction, and negative vorticity is formed over the summit due to squeezing effects of fluid columns. As z_0 decreases the turns are reduced (b-c). For the lowest z_0 the flow is nearly 2D (panel d).

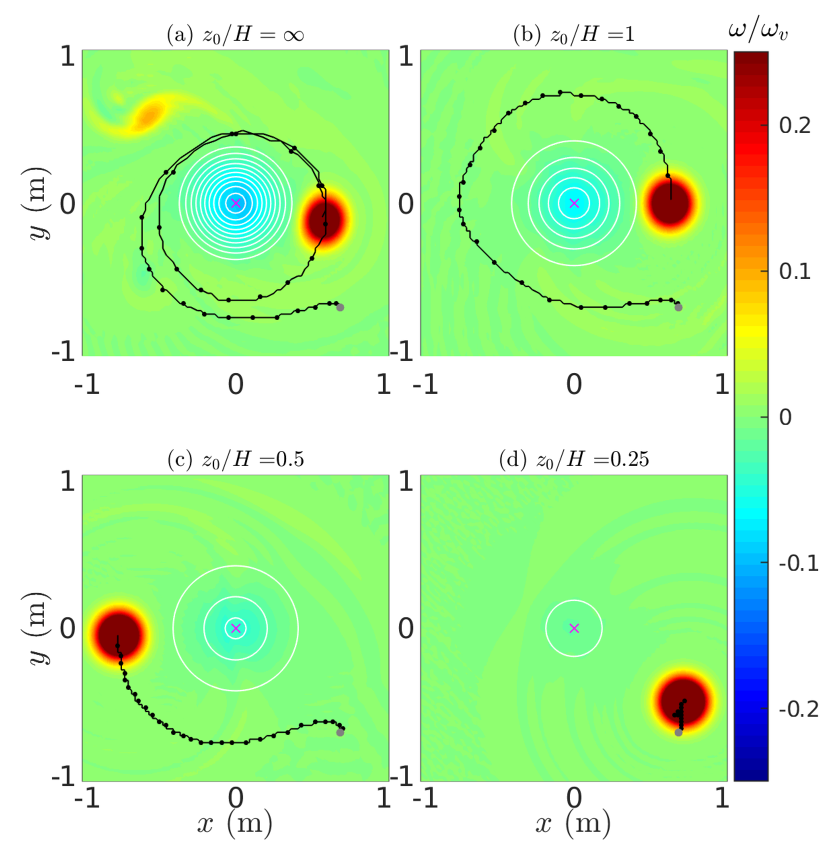


Figure 3: Relative vorticity $\omega_s(x,y)$ at $t/T = 31.5$ in simulations with different z_0.

3 Vortex generation in the Gulf of Mexico

The Campeche cyclone is a semi-permanent mesoscale circulation in the southern Gulf of Mexico (Fig. 4). According to averaged current-meter measurements along the water column, it is one of the few oceanic systems that presents a barotropic vertical structure [6]. The generation of the cyclone under the equivalent-barotropic dynamics is examined.

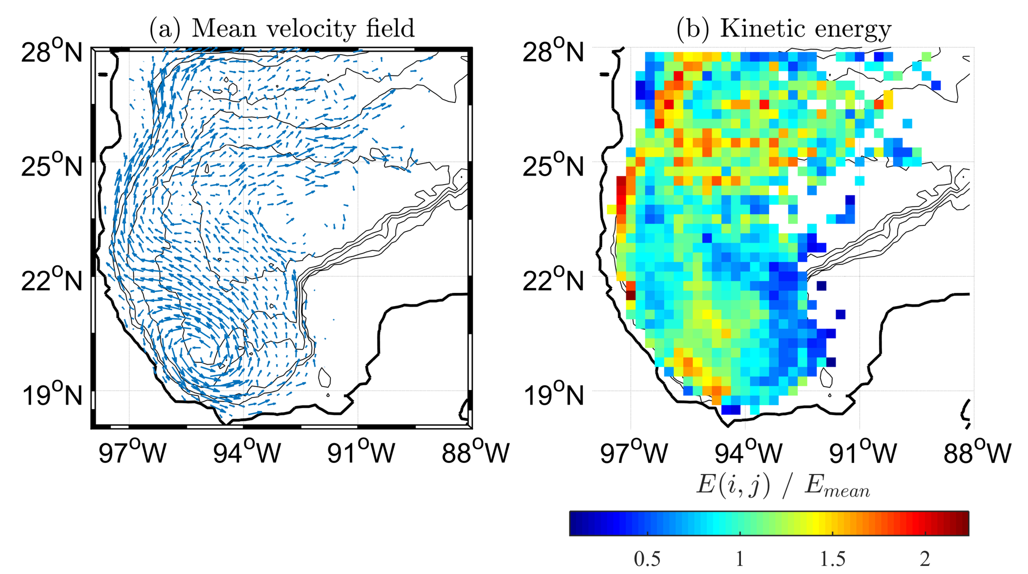


Figure 4: (a) Velocity field in the western Gulf of Mexico calculated from 441 surface drifters during a 7-year period [7]. Maximum value: 0.4 m/s. (b) Kinetic energy per geographical bin (0.25°)

3.1 Simulations using realistic topography

- Western jet-like wind-stress applied over regions with $h > 200$ m (flow over the shelves is minimized).

$$\boldsymbol{\tau}_s = (\tau^x, \tau^y) = \left(-\tau_0 \sin\left[\frac{\pi}{2L}(y + L)\right], 0\right). \quad (7)$$

- Flow starts from rest on a β plane. A smoothed realistic topography is used. Geostrophic contours: $f(y)/F(x,y)$.
- Aim: to investigate whether the formation of a cyclone at the Bay of Campeche is compatible with the equivalent-barotropic dynamics.

The homogeneous and quasi-homogeneous cases (Figs. 5a-c) develop a basin-scale anticyclonic circulation, with no signs of a southern cyclone. For $z_0 = 1000$ m, a cyclonic vortex is formed (d).

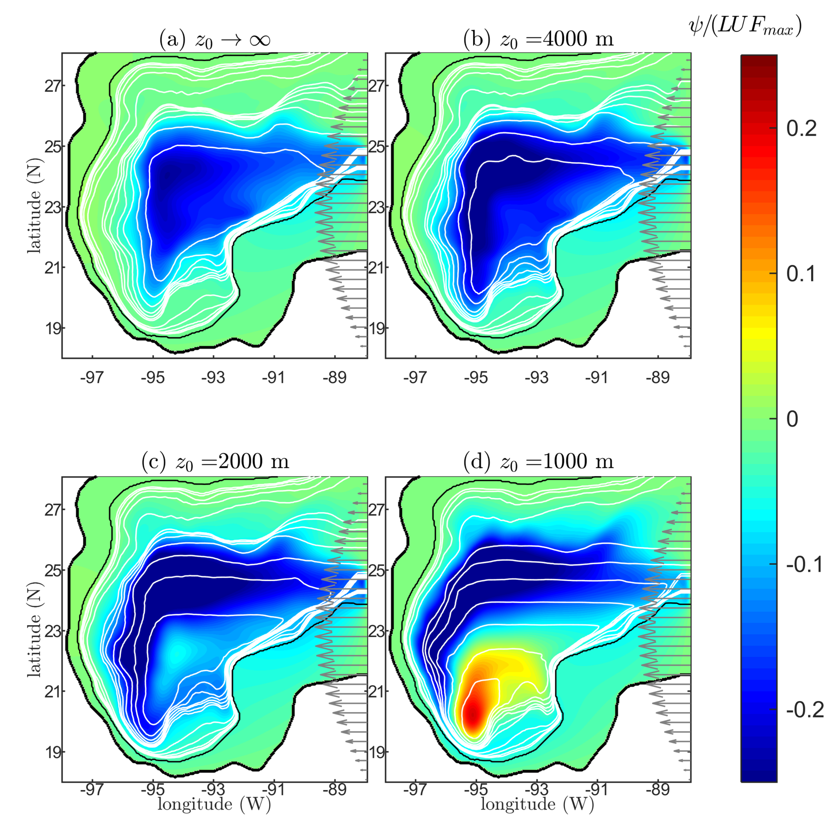


Figure 5: Transport function at day 90 for different z_0. Gray arrows at the eastern side represent the westward wind-stress. Black lines indicate the coast and the 200 m isobath. White curves are $(f_0 + \beta y)/F(x,y)$ contours

3.2 Case with $z_0 = 650$ m

According to mooring observations [6], the average vertical structure in some regions of the Campeche bay is exponential with reference depth $z_0 = 650$ m. The simulation in Fig. 6 displays a cyclonic vortex at the Campeche bay confined by the f/F contours. The maximum speed is 0.14 m/s (in the observations by [6] is 0.36 m/s).

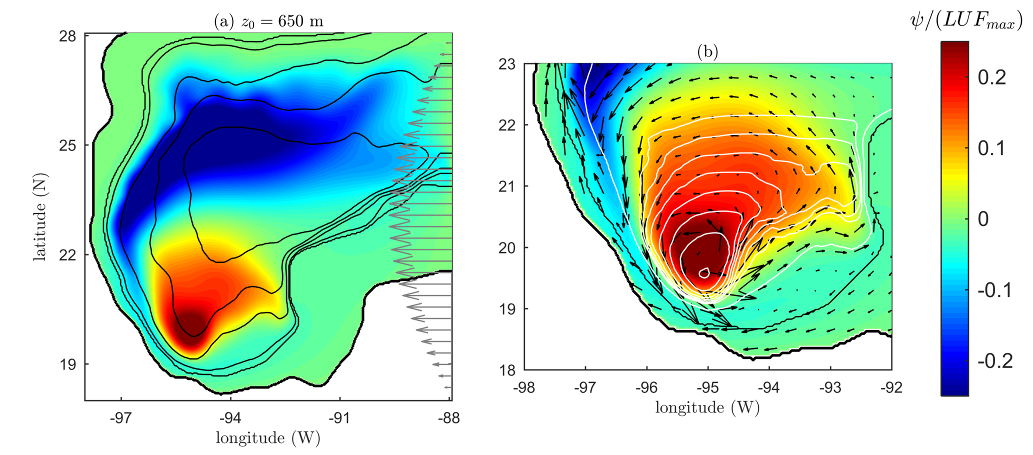


Figure 6: (a) Transport function at day 90 for $z_0 = 650$ m. Black curves are topography contours. (b) Velocity field over the Bay of Campeche. The rms speed is $U = 0.021$ m/s. White contours as in previous figure.

Conclusions 1: the model

- A time-dependent, non-linear equivalent-barotropic model is discussed for studying the effects of variable bottom topography in oceanic flows. The model simulates the vertical structure of stratified flow while maintaining a barotropic character.
- We used an exponential vertical structure $P(z)$, but the formulation admits more general cases as long as

$$\begin{aligned} P(0) &= 1 \\ P(z) &> 0 \quad -h \leq z < 0 \end{aligned} \quad (8)$$

- The error (6) grows for very abrupt $P(z)$.
- Topographic effects are decoupled from the upper region for $z_0 <<$ h (equivalent depth is smoothed and bottom friction reduced).

Conclusions 2,3: the simulations

- The simulations reproduce experimental results of strongly nonlinear vortices in the extreme cases $z_0 >> h$ (Fig. 3a) and $z_0 << h$, *i.e.* nearly 2D (Fig. 3d). Thus, the intermediate cases (b-c) are "validated".
- The model is a useful tool to better understand the formation of an oceanic structure (Fig. 6) under the barotropic dynamics with a vertical structure.

Forthcoming research concerns the use of time-dependent, random forcing together with a large-scale wind in the Campeche cyclone problem. The aim is to identify the prevalence or absence of the cyclonic pattern under different physical conditions.

References

[1] Charney, J.G., 1949. J. Meteorol., 6(6), 372-385.
[2] Killworth, P. D., 1992. J. Phys. Oceanogr., 22, 1379-1387.
[3] Krupitsky *et al.* 1996. J. Phys. Oceanogr., 26(9), 1803-1824.
[4] LaCasce & Isachsen, 2010. J. Phys. Oceanogr., 84, 139-157.
[5] Zavala Sansón *et al.* 2012. J. Fluid Mech., 695, 173-198.
[6] Pérez-Brunius *et al.* 2013. Deep Sea Res. Part II, 85, 182-194.
[7] Zavala Sansón *et al.* 2017. J. Phys. Oceanogr., 47, 387-403.