

An Analytical Model for the Meridional Gradient in CO₂ Forcing

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Abstract

Meridional gradients in CO₂ forcing are known to drive increases in poleward heat transport (Huang and Zhang 2014, Huang et al. 2017). However, the climate factors which control these meridional forcing gradients are not well understood. Building upon the work of Wilson (2012), we build a first-principles, analytical model for CO₂ forcing which requires as input only the temperatures at the surface and roughly 30 hPa, as well as column relative humidity. This model quantitatively captures global variations in clear-sky CO₂ forcing, and shows that the meridional forcing gradient is directly attributable to the meridional surface temperature gradient.

Heterogenous CO₂ forcing from surface-stratosphere temperature contrast

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Introduction

CO₂ forcing varies significantly over the globe, with a strong meridional gradient as well as zonal variations, even in clear-skies (see also Huang 2016):

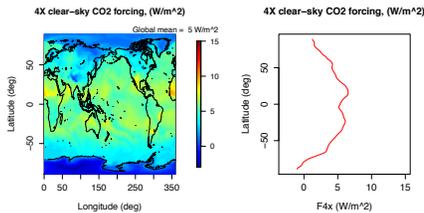
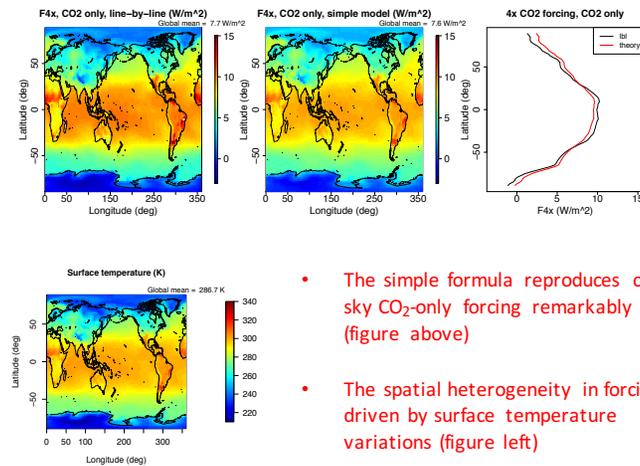


Fig. above: 4X CO₂ forcing, evaluated for a March 1981 snapshot of an AM3 historical run, as calculated line-by-line using RFM (Dudhia 2016).

Research Question: What physics governs these variations? Can we emulate them with a simple model?

Validation for CO₂ only



- The simple formula reproduces clear-sky CO₂-only forcing remarkably well (figure above)
- The spatial heterogeneity in forcing is driven by surface temperature variations (figure left)

Theory

Step 1. Parameterize CO₂ mass absorption coefficients κ (m²/kg) as in Wilson (2012):

$$\kappa(k) = \kappa_0 \exp\left(-\frac{|k - k_0|}{l_k}\right)$$

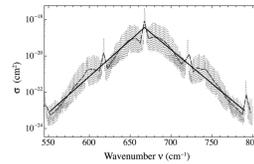


Fig. 1. Absorption cross-section, in cm², for a CO₂ molecule as a function of frequency around 15 μm wavelength (light gray dotted curve); note the logarithmic scale. Also shown are a "course grained" spectrum (medium gray dashed curve) obtained by averaging over intervals of width 5 cm⁻¹, and a drastically simplified version (black, solid line) that we use for the analytical order-of-magnitude estimates. Wilson 2012

Step 2. Calculate optical depth and find emission levels, i.e. levels of unit optical depth, denoted $p_1(k)$:

$$\tau_k(p) = \kappa(k) \int_0^p \frac{p'}{p_s} q \frac{dp'}{g} = \kappa(k) \frac{qp^2}{2gp_s}$$

$$\Rightarrow p_1(k) = \sqrt{\frac{2gp_s}{q\kappa_0}} \exp\left(\frac{|k - k_0|}{2l_k}\right)$$

Step 3. Construct a picture for CO₂ forcing

All orange emission levels exist for both 1x and 4x CO₂. So only change in emission with 4X CO₂ is loss of some surface emission (red) and addition of new stratospheric emission (blue).

⇒ CO₂ forcing only depends on surface-stratosphere temperature contrast!

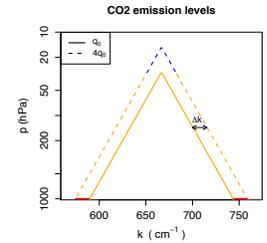


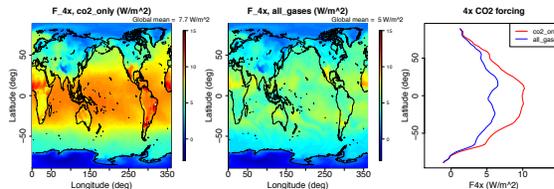
Fig. above: graph of $p_1(k)$ for q_0 corresponding to 280 ppmv

Step 4. Use the above accounting to estimate the CO₂ forcing:

$$F_{4x} = 2l_k \ln 4 \left[\underbrace{\pi B(k_0, T_s)}_{\text{surface}} - \underbrace{\pi B(k_0, T(p_0))}_{\text{stratosphere}} \right] \quad \text{Wilson 2012}$$

Only inputs are surface and stratosphere temperatures!

Future work – H₂O effects



- Figure above shows that effect of H₂O on CO₂ forcing is significant, particularly in tropics where large H₂O path lengths means that H₂O and CO₂ bands overlap
- Meridional gradient in CO₂ forcing strongly dampened by H₂O
- Step 5:** Elaborate on simple model to account for this

Conclusions + References

- We develop a picture for CO₂ forcing based upon the simplified spectroscopy of Wilson (2012).
- The resulting formula is a function of surface-stratosphere temperature contrast only. It predicts spatial variations in CO₂ forcing remarkably well.
- These spatial variations are driven by spatial variations in surface temperature. Water vapor strongly dampens the meridional forcing gradient.

References:

- Dudhia, *The Reference Forward Model (RFM)*, JQSRT 2016
- Wilson and Gea-Banacloche, *Simple model to estimate the contribution of atmospheric CO₂ to the Earth's greenhouse effect*, Am. J. Phys. 2012
- Huang et al., *Inhomogeneous radiative forcing of homogeneous greenhouse gases*, JGR 2016