Physical principles that determine electric streamer parameters

Nikolai G. Lehtinen[1]

[1]University of Bergen

November 22, 2022

Abstract

We propose a new approach to unambiguous determination of parameters of positive and negative electric streamer discharges (streamers). We derive several relations between streamer parameters which allow us to express them in terms of the streamer length L and the external electric field E_e, as functions of streamer radius a. In particular, we find that the streamer velocity V(a) has a maximum at a certain value of radius a_s. We interpret the streamer as a nonlinear instability, whose behavior is determined by maximizing its growth rate, proportional to V. The radius of the streamer is therefore hypothesized to be equal to a_s, which fixes all streamer parameters. Thus, for the first time (to our knowledge) we prove that the streamer parameters are completely determined by E_e and L and suggest a simple way to calculate them that does not require hydrodynamic simulations. The parameters of streamers in air at sea level conditions, calculated in the proposed way, are compared and found to be compatible with experimental data [e.g., velocities obtained by Allen and Mikropoulos, 1999, doi:10.1088/0022-3727/32/8/012 and radii seen on the photographic images of Kochkin et al, 2016, doi:10.1088/0022-3727/49/42/425203], as well as with results of hydrodynamic simulations [Lehtinen and Østgaard, 2018, doi:10.1029/2018JD028646]. We also reproduce the correct values of streamer threshold fields of ~0.45 MV/m for positive streamers and ~0.75-1.25 MV/m for negative streamers.

AE11B-2703: Physical principles that determine electric streamer parameters

Nikolai G. Lehtinen
Nikolai.Lehtinen@uib.no

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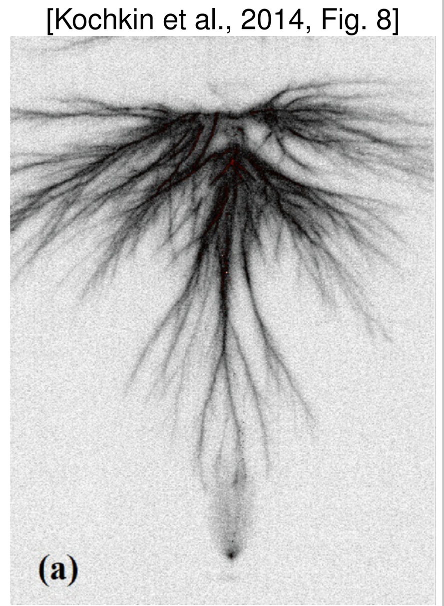
What is a streamer?

Electric streamer discharges are ionized columns in gas (or liquid) which advance by ionizing the material in front of them with the enhanced field at the streamer tip.

[Kochkin et al., 2014, Fig. 8]



Shown here is a laboratory ~MV, 1 m gap discharge, with a complicated branched streamer tree.

Applications:
1. Lightning, sprites
2. Industry (suprathermal electrons)
3. In the future: x-ray production (still unexplained)

Streamer system is
- highly non-uniform
- thin streamers, 10^{-4}–10^{-2} m [Bazelyan and Raizer, 1998, p. 47]
- branching
- extinguishing and re-igniting
- reverse streamers (negative)
- stepping (negative)

Streamer mechanism [Loeb and Meek, 1941]

1. Electron drift (forward for negative streamer)
2. Electron diffusion (if no photoionization)
3. Photoionization (the main mechanism in air)
   Photons with wavelength between 980 Å and 1025 Å are produced by electron collisions with N_2 which then travel ahead of the streamer and ionize O_2.

$$e^- + N_2 \to e^- + N_2^*, \qquad N_2^* \to N_2 + h\nu(980\text{–}1025\ \text{Å}), \qquad h\nu + O_2 \to O_2^+ + e^-$$

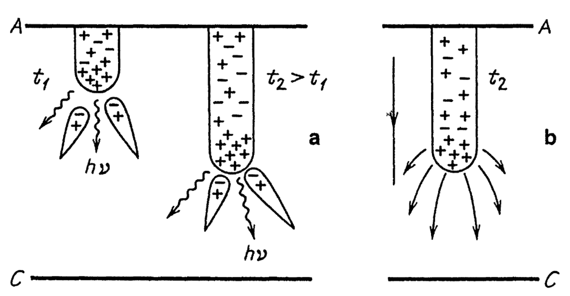


Figure 1: Positive streamer [Raizer, 1991, p. 335]

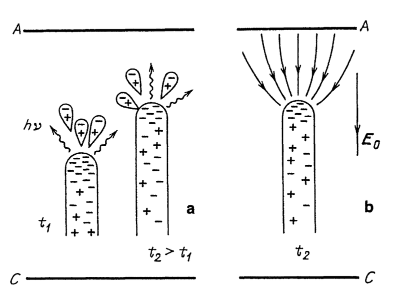


Figure 2: Negative streamer [Raizer, 1991, p. 338]

Note: the avalanches started by photoelectrons are directed outward, but the streamer moves so fast that it catches up with them.

The goal
To identify physical principles that determine and to calculate streamer **radius, speed** and other parameters (e.g., field in front of the streamer and inside the channel)

This task had not been solved before now. Another important unsolved task in streamer physics is understanding the physical principles of streamer **branching**.

Hydrodynamic simulations

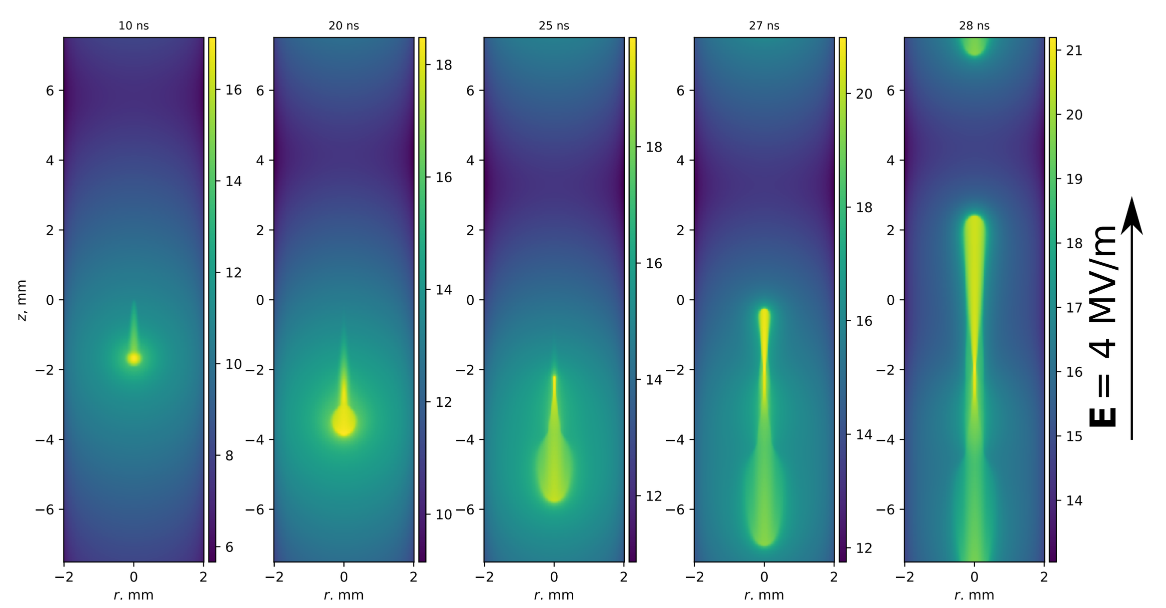


Figure 3: Evolution of $\log_{10} n_e$ in the avalanche-to-streamer transition. [Lehtinen and Østgaard, 2018]

They give a quantitative answer, but ...
- offer no insight into the physical reasons for the parameters;
- possibly have numerical errors such as instabilities and numerical diffusion:
  ... the authors [have] disagreement on whether the branching of model streamers ... is a consequence of a numerical instability. — Pasko [2006, p. 291]

We prefer to understand the physical principles that quantitatively determine streamer propagation (and branching) than to rely blindly on possibly incorrect computer calculations.

Flat front perturbations

- field at the protrusions increases, perturbations grow
- fastest-growing perturbation survives, determines transverse size
- **electron drift + diffusion:** this size is $a \propto D^{1/4}$ [Derks et al., 2008]
- no theory yet with **photoionization**

Other proposed explanations

The following hypothesis were proposed:
- **Electron diffusion** causes transverse spreading of the streamer and therefore determines radius. Produces a radius that is too small [Dawson and Winn, 1965; Gallimberti, 1972; Qin and Pasko, 2014].
- Streamer **"chooses" the field** at the tip at which it "wants" to propagate [Bazelyan and Raizer, 1998, p. 277] (which also determines all other parameters).
  The physical reason why this particular field should be chosen is not apparent.

Streamer model and relations between parameters

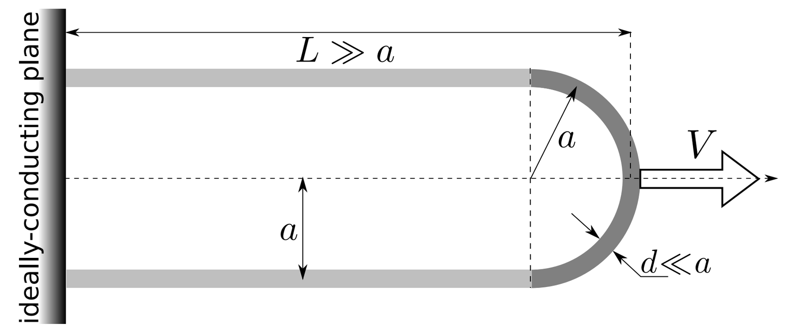


Figure 4: The streamer model (sizes)

Sizes:
- Streamer is a cylinder (**channel**) with a hemispherical cap (**head**)
- Length L (given!) $\quad L = \int V\,dt$
- Radius $a \ll L$
- Thickness $d \ll a$

Equation 1: Fields

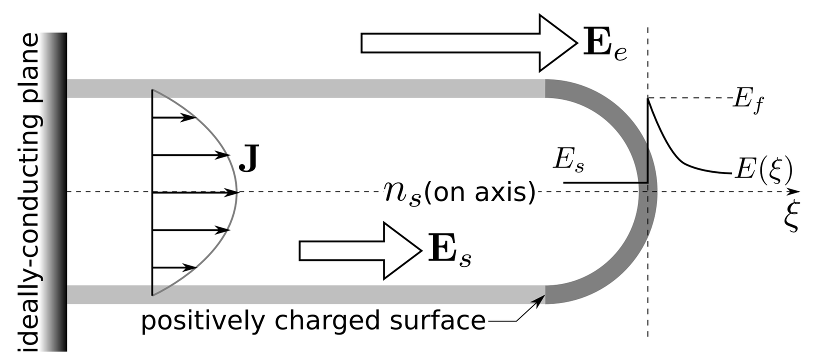


Figure 7: The streamer model (fields)

- External E_e (given!)
- Inside $E_s \ll E_e$ due to high conductivity, all charges are at surface
- Still $E_s > 0$ because there is a current in the channel $\propto n_s$.
- Just outside $E_f \gg E_e$

An electrostatic model (solved using the **method of moments** or **MoM**) determines relation between E_e, E_s, E_f (but E_s and E_f separately are still uknown!).

Equation 2: Currents

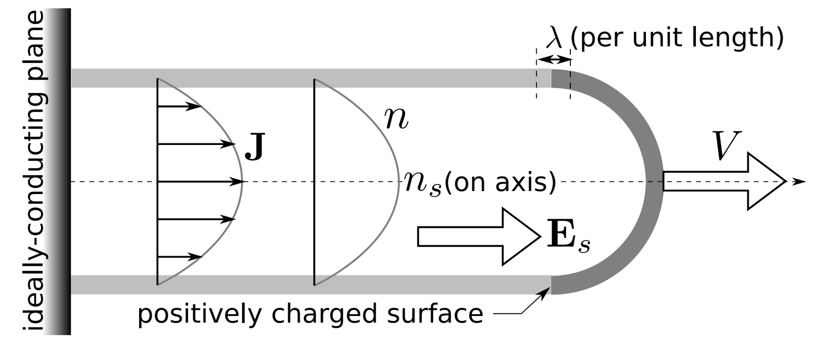


Figure 8: The streamer model (currents)

- Charge on the surface per unit length λ is from MoM and E_s
- The total current is $I = \lambda V$
- It is also calculated from n_s and E_s as $I = \int \mathbf{J}\,dA_\perp$

Thus, we have a relation between E_s, n_s, V.

Equation 3: Ionization front

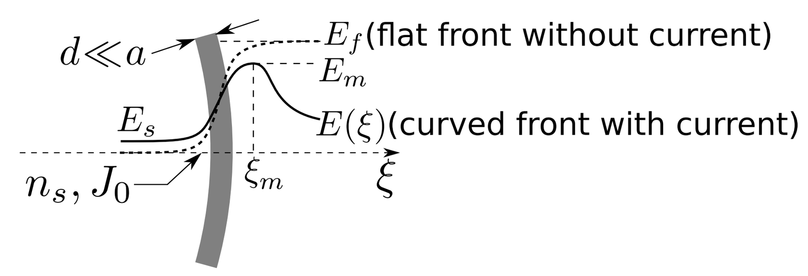


Figure 11: The streamer model (the front)

The flat front theory is used to relate n_s to E_f. We also have corrections to this theory:
- to include the current J_0 (on the axis)
- to include curvature
- maximum field is not E_f but corrected value E_m (depending on d)

Equation 4: Photoionization

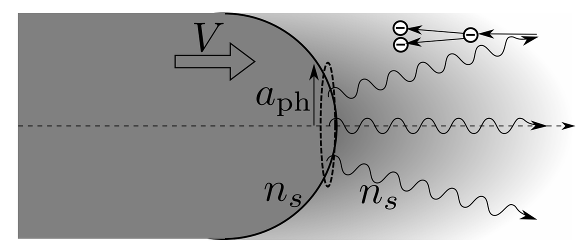


Figure 12: The streamer model (photoionization)

- Ionizing photons are produced in the front $\propto$ ionization rate
- Photon production volume (and the number) $\propto \pi a_{ph}^2$, $a_{ph} \sim a$
- Ionization occurs remotely, described by Zheleznyak et al. [1982]
- Electron avalanche has length d in streamer frame, which depends on V and E_f
- The electron density in the end of avalanche must match n_s

Thus, we relate V, E_f and $a_{ph} \sim a$.

System of equations for E_s, E_f, a, n_s, V

1. An electrostatic model determines relation between E_e and E_s, E_f
2. The ionization n_s is related to velocity V: the current flowing in the streamer channel ($\propto n_s$) is equal to "flying charged rod" current $\propto V$
3. The ionization n_s is related to E_f by the results of the flat front theory [Lagarkov and Rutkevich, 1994].
4. Velocity V is related to the radius a (and field E_f) through the photoionization mechanism [Naidis, 2009; Pancheshnyi et al., 2001].

We notice:
- # unknowns = # constraints + 1
- it is convenient to choose the streamer radius a as a free parameter
- velocity V and other parameters are functions of a
- everything is also a function of **laboratory-condition fixed E_e and L**

Maximum velocity hypothesis

The number of these constraints is, however, smaller by one than the number of unknowns. This means that we still cannot fix all the streamer parameters based only on E_e and L, and the streamer radius a is still a free parameter. However, we can express V and other parameters as functions of a. All parameters except V are monotonous functions of a. But $V(a)$ has a maximum!

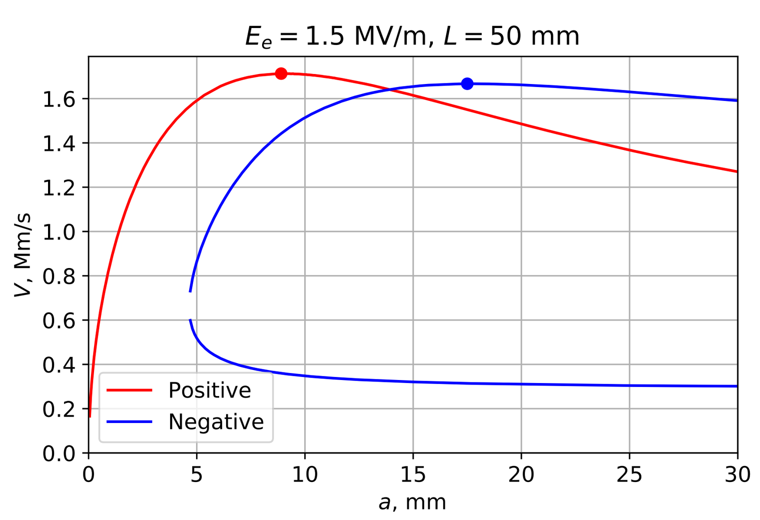


Figure 13: An example of solutions for $V(a)$ showing one solution for positive and two or zero solutions for negative streamers in air. The large dot shows the value at which V is maximized.

Intuitively: The velocity drops for narrow streamers because there is not enough photon production, and for wide streamers because the field enhancement at its tip becomes insufficient.
- in flat-front perturbation theory, the surviving transverse size was the fastest-growing instability
- by analogy, choose the maximum V (and the corresponding radius value a_s) as the correct parameter values
- thus, the streamer is treated as a **nonlinear instability**

Equation 5
We have identified one more constraint: $V \to \max$.
Now, # unknowns = # constraints and we can unambiguously determine all streamer parameters.

The calculated parameters for lab conditions

We compare to experimental result of Allen and Mikropoulos [1999].

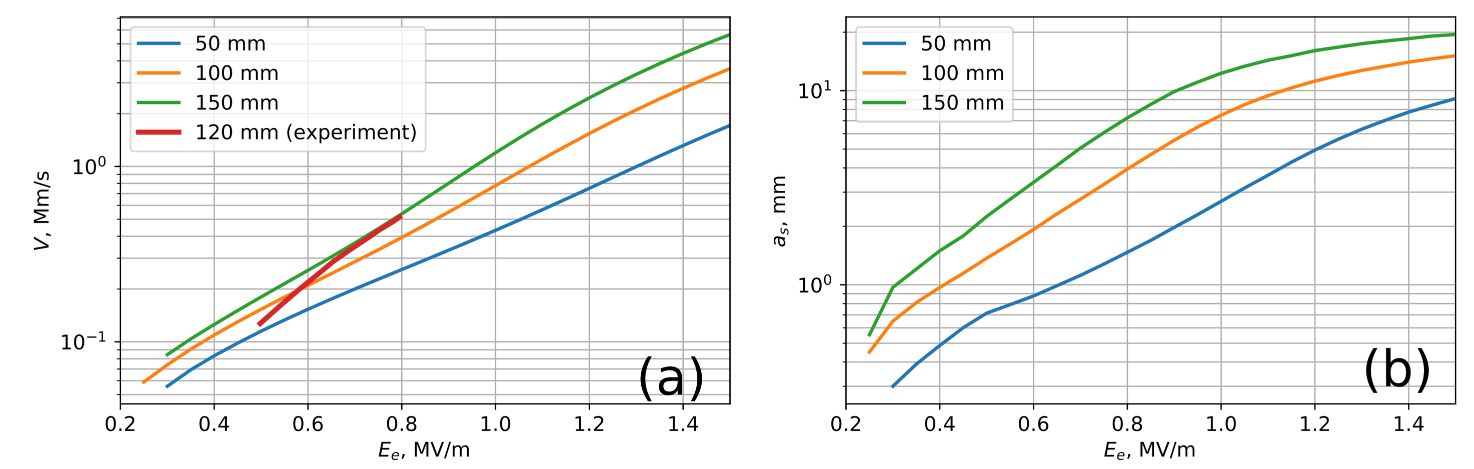


Figure 14: Results for positive streamers as functions of external field E_e, for three different values of L = 50, 100 and 150 mm: (a) velocity V, (b) radius a_s. The measurements of Allen and Mikropoulos [1999, Fig. 10] at L = 120 mm are presented together with calculated V results in panel (a).

Negative streamers

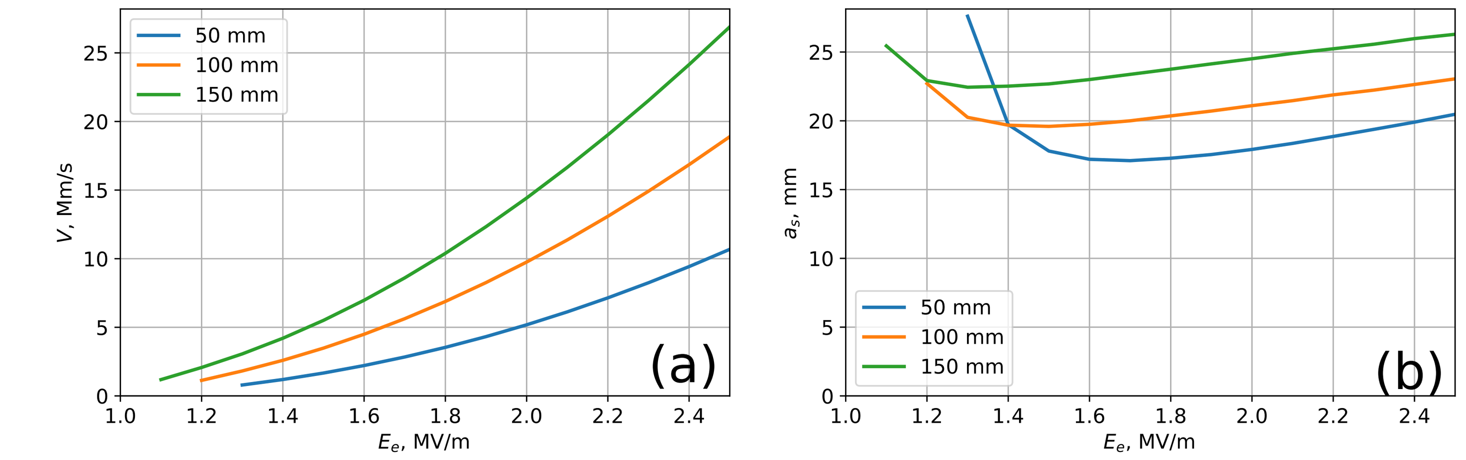


Figure 15: Results for negative streamers (same lab conditions).

Streamer threshold fields [Raizer, 1991, p. 362]

Interestingly, the physical reason for the minimum external field $E_e = E_{\pm t}$ at which the streamer still can propagate is different for different streamer polarities.

Positive streamers: As the external field E_e decreases, the internal field E_s also decreases, and the three-body attachment becomes increasingly important. The number of attachment lengths determines the threshold $E_{+t} \approx 0.45$ MV/m. The threshold is **lower** for shorter streamers.

Negative streamers: $a \to \infty$ at threshold $E_{-t} \approx$ 0.75–1.25 MV/m. The threshold is **higher** for shorter streamers.

The predicted dependence on streamer length may be checked experimentally.

Summary

We identified the physical principles and found a way to calculate streamer parameters, such as its velocity and radius, on the basis of the given external electric field and the streamer length.

Possible future directions:
1. More comparisons, with both experimental data and hydrodynamic simulations.
2. Hydrodynamic simulation study of the parameter stability.
   I.e., try to vary the parameters and see if they come back to the fixed value.
3. Nonuniform fields and branching.
   The presented theory produces no branching in uniform fields.
4. What happens when the streamer becomes too wide (nothing/branching/something else)?

Acknowledgements

This study was supported by the European Research Council under the European Union's Seventh Framework Programme (FP7/2007-2013)/ERC grant agreement n. 320839 and the Research Council of Norway under contracts 208028/F50, 216872/F50 and 223252/F50 (CoE).

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