

Radiative Forcing and Climate Sensitivity

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November 30, 2022

Abstract

Current estimates of the impact of an increase in greenhouse gas concentrations on global warming, including by the IPCC and in General Circulation Models, are based on radiative forcing. Two recently published formulations of the theoretical foundation for radiative forcing are reviewed. Radiative forcing at the tropopause is calculated by assuming that the absorption of terrestrial radiation by greenhouse gases is determined by their spectral properties, using a radiative transmittance function based on the line strength and line shape of the absorption lines and the vertical optical mass, whilst, under conditions of local thermodynamic equilibrium, the emission of radiation at each layer of the atmosphere is given by the Planck blackbody function at the local atmospheric temperature. Radiative forcing is given by the net change in radiative flux at the troposphere due to an increase in greenhouse gases. Climate change is seen to take place when the system responds to restore the radiative equilibrium. Without any theoretical foundation, a linear relationship between the change in surface temperature in °C and radiative forcing is assumed. Here, IPCC 2013's estimate of radiative forcing of 2.83 W/m² due to the increase in greenhouse gases from 1750 to 2011 is used to calculate the resulting change in radiative flux at the Earth's surface under reasonable assumptions, and the Stefan-Boltzmann law is applied to calculate the change in surface temperature of between 0.8 and 1.0 °C. This represents a climate sensitivity of around 0.32 °C/(W/m²), about one third of the climate sensitivity of 1.0 °C/(W/m²) used by IPCC 2013 that was obtained from the mean regression-based values of 30 climate models.

Radiative Forcing and Climate Sensitivity †

† Extract from Underwood (2018c). *Radiative Forcing and Climate Sensitivity. Unpublished*

Current estimates of the impact of increases in greenhouse gas concentrations on global warming, including in General Circulation Models (GCMs), are based on radiative forcing. There are various definitions of “radiative forcing” but that adopted by the Intergovernmental Panel on Climate Change (IPCC) is the change in net downward radiative flux (shortwave solar less longwave thermal flux) at the tropopause after allowing for stratospheric temperatures to readjust to radiative equilibrium, while holding surface and tropospheric temperatures and state variables such as water vapor and cloud cover fixed at the unperturbed values (Myhre et al., 2013). Climate change is seen to take place when the system responds to restore the radiative equilibrium at the top of the atmosphere, e.g. through an increase in surface temperature, to offset a decrease in outgoing thermal radiation at the top of the atmosphere resulting from an increase in absorption by greenhouse gases in the atmosphere. The first IPCC report on climate change issued in 1990 described its use of radiative forcing to measure the relative contributions of each agent, in particular greenhouse gases, rather than climate change itself, on grounds that they could estimate the former with a great more certainty than the latter. This was seen in the context of policy formulation where the relative importance of the agents was considered of major significance in assessing the effectiveness of response strategies, without resolving whether any warming actually occurred (Shine et al., 1990).

“Climate sensitivity” to radiative forcing is the increase in surface temperature required to offset the radiative forcing represented by the reduction in emissivity at the top of the atmosphere. The IPCC bases its estimations of potential increases in surface temperature with increases in concentrations of greenhouse gases on the assumption of a linear relationship between radiative forcing, ΔF , and the subsequent change in equilibrium surface temperature (ΔT_s) for each gas, $\Delta T_s = \lambda \Delta F$, where λ is the climate sensitivity, usually with units in $^{\circ}\text{C}/(\text{W}/\text{m}^2)$.

The concept of radiative forcing derives from analysis on the effect of greenhouse gases on the pressure dependence of radiative heat transfer in the atmosphere developed separately by Lewis Kaplan and Gilbert Plass in the 1950s (Kaplan, 1952; Plass, 1956). These calculations assumed blackbody radiation at the Earth’s surface and upward and downward blackbody radiation from each layer of the atmosphere. Two recently published formulations of the theoretical basis for radiative forcing as a measure of the impact of an increase in greenhouse gases on the surface temperature of the Earth are reviewed.

Mlynczak et al. (2016)

Mlynczak et al. (2016) calculates radiative forcing from the change in net radiative flux at the tropopause due to an increase in the concentration of greenhouse gases. The net radiative flux F_{net} at altitude u is given by $F_{\text{net}}(\mathbf{u}) = F_{-}(\mathbf{u}) - F_{+}(\mathbf{u})$, where the upwelling and downwelling fluxes F_{+} and F_{-} are obtained by angular and spectral integration of the upwelling and downwelling monochromatic radiances $I_{\nu}(\mu)$ and $I_{\nu}(-\mu)$ over a hemisphere: $F_{+}(u) = -2\pi \int_0^{\infty} \int_0^1 I_{\nu}(+\mu, u) \mu d\mu d\underline{\nu}$, and $F_{-}(u) = 2\pi \int_0^{\infty} \int_0^1 I_{\nu}(-\mu, u) \mu d\mu d\underline{\nu}$, so $F_{\text{net}}(\mathbf{u}) = 2\pi \int_0^{\infty} \int_0^1 I_{\nu}(-\mu, u) \mu d\mu d\underline{\nu} - 2\pi \int_0^{\infty} \int_0^1 I_{\nu}(+\mu, u) \mu d\mu d\underline{\nu}$, where $\underline{\nu} = 1/\lambda$ is the wavenumber, λ is the wavelength, and μ is the zenith direction cosine. The radiance (specific intensity) at altitude u , $I_{\nu}(\mu, u)$, is obtained by solving the fundamental equations of radiative transfer.

The upward radiance at altitude u is given by the sum of the radiative flux from the Earth’s surface times the transmittance between the Earth’s surface and altitude u and the integral between the surface and altitude u of the radiative flux emitted by the atmosphere at altitude u' times the change in the transmittance between altitudes u and u' , assuming blackbody radiation $B_{\nu}(T)$ at the Earth’s surface and local thermodynamic equilibrium in the atmosphere, under which the emission of radiation at each layer of the atmosphere is given by the Planck blackbody function at the local atmospheric temperature, $I_{\nu}(+\mu, u) = B_{\nu}(T_s) \tau_{\nu}(u, 0) + \int_0^u B_{\nu}(T(u')) [\partial \tau_{\nu}(u, u') / \partial u'] du'$, where T_s and T are the surface and atmospheric temperatures, the energy flux density $B_{\nu}(T) = (2\pi h c \underline{\nu}^3) / [\exp(h c \underline{\nu} / k T) - 1]$ is the Planck blackbody function, which describes the energy distribution of a black body at temperature T , and $\tau_{\nu}(u, u')$ is the transmittance of the atmosphere between altitudes u and u' .

The downward radiative flux is given by the integral between altitude u and the tropopause of the radiative flux emitted by the atmosphere at altitude u' times the change in the transmittance between altitudes u and u' , $I_{\nu}(-\mu, u) = - \int_u^{\infty} B_{\nu}(T(u')) [\partial \tau_{\nu}(u, u') / \partial u'] du'$.

The transmittance function, which measures the fraction of incident radiation that passes through this layer of the atmosphere, is given by $\tau_{\nu}(u, u') = \exp[-\sum_i S_i(T) g_i(\underline{\nu} - \underline{\nu}_0) m(u, u') / \mu]$, where $S_i(T)$ is the i th line strength at temperature T , $g_i(\underline{\nu} - \underline{\nu}_0)$ is the normalized line shape of the i th line, and $m(u, u')$ is the vertical optical mass between altitudes u and u' . The line shape function is dependent on pressure and temperature.

Radiative forcing is defined as the *change* in the net radiative flux at the tropopause due to an increase in greenhouse gas.

Mlynczak, M. G. et al. (2016) The spectroscopic foundation of radiative forcing of climate by carbon dioxide. *Geophys. Res. Lett.*, 43, 5318–5325. <https://doi.org/10.1002/2016GL068837>

Schwartz (2018)

Schwartz (2018) calculates radiative forcing from the difference between absorption and emission. For any absorption line or band the volumetric rate of absorption of upwelling radiant energy $P_{\text{abs(up)}}$ at altitude u is equal to the product of the absorption coefficient times the incident irradiance integrated over the frequency range $d\nu$ for which there is appreciable absorption, $P_{\text{abs(up)}}(\nu, u) = n_i(\mathbf{u}) \int B_{s,\nu}(\nu', T_s) \sigma(\nu', u) d\nu'$, where $n_i(\mathbf{u})$ is the number concentration of absorbing molecules of species X_i and where $\sigma(\nu, u)$ is the absorption cross section of the molecule, and the energy flux density is given by the Planck function, $B_{s,\nu}(\nu, T_s) = (2\pi h \nu^3 / c^2) / [\exp(h\nu / k T_s) - 1]$, which describes the energy distribution of a black body at the surface temperature T_s . The Planck function is assumed to vary slowly with frequency over a given absorption band, so $P_{\text{abs(up)}}(\nu, u) = B_{s,\nu}(\nu', T_s) n_i(\mathbf{u}) \int \sigma(\nu', u) d\nu'$, or denoting the integral $\int \sigma(\nu', u) d\nu' = S_{\nu}(\nu, u)$, $P_{\text{abs(up)}}(\nu, u) = B_{s,\nu}(\nu, T_s) S_{\nu}(\nu, u) n_i(\mathbf{u})$, where $S_{\nu}(\nu, u)$ is the band strength or integrated absorption coefficient.

Assuming that the altitude dependence of the band strength may be neglected, the total absorption of upwelling irradiance per area in an atmospheric column is given by the integral of $P_{\text{abs(up)}}(\nu, u)$ over the height of the column, $F_{\text{abs}}(\nu) = B_{s,\nu}(\nu, T_s) S_{\nu} \int n_i(\mathbf{u}) du$. For a well-mixed gas, the number concentration of absorbing molecules of species X_i , $n_i(\mathbf{u}) = x_i n_{\text{air}}(\mathbf{u})$, where x_i is the mixing ratio and n_{air} is the number concentration of air molecules, so $F_{\text{abs}}(\nu) = B_{s,\nu}(\nu, T_s) S_{\nu} x_i \int n_{\text{air}}(\mathbf{u}) du$, where $\int n_{\text{air}}(\mathbf{u}) du$ is the number of molecules of air per area in the atmospheric column.

The emission is assumed to be equal to the absorption of upwelling radiant energy reduced by the Boltzmann factor, $\exp[-h\nu(T(u)^{-1} - T_s^{-1})/k]$, where $T(u)$ is the local atmospheric temperature, a function of altitude. At a given altitude u in the atmosphere the volumetric rate of emission is $P_{\text{emit(down)}}(\nu, u) = P_{\text{emit(up)}}(\nu, u) = B_{s,\nu}(\nu, T_s) x_i S_{\nu}(\nu, u) n_{\text{air}}(\mathbf{u}) \exp[-h\nu(T(u)^{-1} - T_s^{-1})/k] / 2$, where the Boltzmann factor governing the emitted power adjusts for the local atmospheric temperature¹.

Then the local volumetric heating rate is the local difference between absorption and emission $P_{\text{net}}(\nu, u) = P_{\text{abs(up)}}(\nu, u) - P_{\text{emit(down)}}(\nu, u) - P_{\text{emit(up)}}(\nu, u)$, and the cumulative change in upwelling irradiance from the surface to altitude u due to a specific absorption band is obtained from the vertical integral of the difference in absorbed and upward emitted power, $F_{\text{net(up)}}(\nu) = \int_0^u [P_{\text{abs(up)}}(\nu, u') - 2P_{\text{emit(up)}}(\nu, u')] du'$, so $F_{\text{net(up)}}(\nu) = B_{s,\nu}(\nu, T_s) S_{\nu}(\nu, u) x_i \int_0^u n_{\text{air}}(\mathbf{u}') [1 - \exp[-h\nu(T(u')^{-1} - T_s^{-1})/k]] du'$, and the net top of atmosphere forcing is given by the integral over the entire height of the vertical column.

Schwartz, S. E. (2018). The Greenhouse Effect and climate change. <https://doi.org/10.1002/essoar.81ea1b43594141c6.558e238c20a84445.1>

1. Schwartz (2018) assumes that there is no contribution from downwelling irradiance, but the doubling up of the emission results from considering the change in the net flux arising from absorption and upward and downward emissions at layer du and then integrating from the surface to altitude u rather than calculating the contributions to the change in the net upward flux at du from sum of the integral of absorption and upward emission by the atmosphere from the surface to altitude u and the integral of downward emission from altitude u to the top of the atmosphere, then integrating over the whole column. Consequently it also omits the contribution to the net forcing from downward emission from above du .

Mlynczak et al. (2016) and Schwartz (2018) calculate radiative forcing at the tropopause by assuming that the absorption of blackbody terrestrial radiation by greenhouse gases is determined by the surface temperature at which the radiation was originally emitted and the spectral properties of greenhouse gases, using a radiative transmittance function based on the line strength and line shape of the absorption lines and the vertical optical mass, whilst, under conditions of local thermodynamic equilibrium arising from predominately collisional deactivation of excited greenhouse gas molecules up to 60 km altitude, the emission of radiation at each layer of the atmosphere is given by the Planck blackbody function at the local atmospheric temperature, which decreases with height in the troposphere. Radiative forcing is given by the net change in radiative flux at the troposphere due to an increase in greenhouse gases, which is seen to create a transfer of radiative energy to the atmosphere and radiative imbalance at the tropopause. A linear relationship between the change in surface temperature in $^{\circ}\text{C}$ and radiative forcing is assumed.

If the daily cycle is followed, during which solar radiation is absorbed by the Earth’s surface and atmosphere and is emitted, then partly absorbed by the atmosphere, re-emitted equally up and down, partly re-absorbed by the Earth’s surface, re-emitted, etc, until all of the solar energy absorbed by the Earth’s surface and atmosphere has been re-emitted at the top of the atmosphere, radiative balance is restored. The surface temperature reflects the blackbody equivalent of the photon flux density of the emission from the Earth’s surface.

A change in greenhouse gas concentration over time, of any significance over a number of years, will affect the amount of terrestrial radiation absorbed and re-emitted on a daily basis by the atmosphere until the spectra within the greenhouse gas absorption bands are fully absorbed. This determines a maximum surface temperature for a given solar irradiance. The Greenhouse Effect occurs because half of the radiation is emitted upward and half downward, adding to the total infrared radiation emitted by the surface during the daily cycle, and increasing the temperature at the Earth’s surface (Underwood, 2018a, 2018b).

It is less easy to relate the net change in radiative flux at the troposphere due to an increase in greenhouse gases to a change in the average daily surface temperature. If it is assumed that the upwelling flux at the tropopause is equal to the flux at the top of the atmosphere, and the change in radiative flux at the tropopause due to an increase in greenhouse gases creates a proportionate change in terrestrial radiation absorbed by the atmosphere, or a proportionate change in the flux emitted by the Earth’s surface, the estimated radiative forcing of 2.83 W/m^2 due to the increase in greenhouse gases from 1750 to 2011 (Myhre et al., 2013) would result in an increase in the flux emitted by the surface of between 4.3 and 4.9 W/m^2 and, applying the Stephan-Boltzmann Law, an increase in surface temperature of between 0.8 and 1.0 $^{\circ}\text{C}$. This represents a climate sensitivity of around 0.32 $^{\circ}\text{C}/(\text{W}/\text{m}^2)$, about one third of the climate sensitivity of 1.0 $^{\circ}\text{C}/(\text{W}/\text{m}^2)$ used by IPCC 2013 (Flato et al., 2013) that was obtained from the mean regression-based value of 30 climate models, and 30% less than the climate sensitivity of 0.43 $^{\circ}\text{C}/(\text{W}/\text{m}^2)$ used by Plass (1956) and Kaplan (1960).

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Mlynczak et al. (2016) calculates radiative forcing from the change in net radiative flux at the tropopause due to an increase in the concentration of greenhouse gases. The net radiative flux F_{net} at altitude u is given by

$$\mathbf{F}_{\text{net}}(\mathbf{u}) = \mathbf{F}_{-}(\mathbf{u}) - \mathbf{F}_{+}(\mathbf{u}),$$

where the upwelling and downwelling fluxes F_{+} and F_{-} are obtained by angular and spectral

integration of the upwelling and downwelling monochromatic radiances $I_{\underline{\nu}}(\mu)$ and $I_{\underline{\nu}}(-\mu)$ over a hemisphere, $F_+(u) = -2\pi \int_0^{\infty} \int_0^1 I_{\underline{\nu}}(+\mu, u) \mu d\mu d\underline{\nu}$, and $F_-(u) = 2\pi \int_0^{\infty} \int_0^1 I_{\underline{\nu}}(-\mu, u) \mu d\mu d\underline{\nu}$, so

$$\mathbf{F}_{\text{net}}(\mathbf{u}) = 2\pi \int_0^{\infty} \int_0^1 I_{\underline{\nu}}(-\mu, u) \mu d\mu d\underline{\nu} - 2\pi \int_0^{\infty} \int_0^1 I_{\underline{\nu}}(+\mu, u) \mu d\mu d\underline{\nu},$$

where $\underline{\nu} = 1/\lambda$ is the wavenumber, λ is the wavelength, and μ is the zenith direction cosine. The radiance (specific intensity) at altitude u , $I_{\underline{\nu}}(\mu, u)$, is obtained by solving the fundamental equations of radiative transfer.

The upward radiance at altitude u is given by the sum of the radiative flux from the Earth's surface times the transmittance between the Earth's surface and altitude u and the integral between the surface and altitude u of the radiative flux emitted by the atmosphere at altitude u' times the change in the transmittance between altitudes u and u' , assuming blackbody radiation $B_{\underline{\nu}}(T)$ at the Earth's surface and local thermodynamic equilibrium in the atmosphere, under which the emission of radiation at each layer of the atmosphere is given by the Planck blackbody function at the local atmospheric temperature,

$$I_{\underline{\nu}}(+\mu, u) = B_{\underline{\nu}}(T_s) \tau_{\underline{\nu}}(u, 0) + \int_0^u B_{\underline{\nu}}(T(u')) [\partial \tau_{\underline{\nu}}(u, u') / \partial u'] du',$$

where T_s and T are the surface and atmospheric temperatures, the energy flux density $B_{\underline{\nu}}(T) = (2\pi h c \underline{\nu}^3) / [\exp(hc\underline{\nu}/kT) - 1]$ is the Planck blackbody function, which describes the energy distribution of a black body at temperature T , and $\tau_{\underline{\nu}}(u, u')$ is the transmittance of the atmosphere between altitudes u and u' .

The downward radiative flux is given by the integral between altitude u and the tropopause of the radiative flux emitted by the atmosphere at altitude u' times the change in the transmittance between altitudes u and u' ,

$$I_{\underline{\nu}}(-\mu, u) = - \int_u^{\infty} B_{\underline{\nu}}(T(u')) [\partial \tau_{\underline{\nu}}(u, u') / \partial u'] du'.$$

The transmittance function, which measures the fraction of incident radiation that passes through this layer of the atmosphere, is given by

$$\tau_{\underline{\nu}}(u, u') = \exp[- \sum_i S_i(T) g_i(\underline{\nu} - \underline{\nu}_0) m(u, u') / \mu],$$

where $S_i(T)$ is the i th line strength at temperature T , $g_i(\underline{\nu} - \underline{\nu}_0)$ is the normalized line shape of the i th line, and $m(u, u')$ is the vertical optical mass between altitudes u and u' . The line shape function is dependent on pressure and temperature.

Radiative forcing is defined as the *change* in the net radiative flux at the tropopause due to an increase in greenhouse gas.

Schwartz (2018)

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For any absorption line or band the volumetric rate of absorption of upwelling radiant energy $P_{\text{abs(up)}}$ at altitude u is equal to the product of the absorption coefficient times the incident irradiance integrated over the frequency range $d\nu$ for which there is appreciable absorption,

$$P_{\text{abs(up)}}(\nu, u) = n_i(u) \int B_{s,\nu'}(\nu', T_s) \sigma(\nu', u) d\nu',$$

where $n_i(u)$ is the number concentration of absorbing molecules of species X_i and where $\sigma(\nu, u)$ is the absorption cross section of the molecule, and the energy flux density is given by the Planck function, $B_{s,\nu}(\nu, T_s) = (2\pi h \nu^3 / c^2) / [\exp(h\nu/kT_s) - 1]$, which describes the energy distribution of a

black body at the surface temperature T_s . The Planck function is assumed to vary slowly with frequency over a given absorption band, so $P_{\text{abs(up)}}(\nu, u) = B_{s,\nu'}(\nu', T_s) n_i(u) \int \sigma(\nu', u) d\nu'$, or denoting the integral $\int \sigma(\nu', u) d\nu' = S_\nu(\nu, u)$, gives

$$\mathbf{P}_{\text{abs(up)}}(\nu, \mathbf{u}) = \mathbf{B}_{s,\nu}(\nu, T_s) \mathbf{S}_\nu(\nu, \mathbf{u}) n_i(\mathbf{u}),$$

where $S_\nu(\nu, u)$ is the band strength or integrated absorption coefficient.

Assuming that the altitude dependence of the band strength may be neglected, the total absorption of upwelling irradiance per area in an atmospheric column is given by the integral of $P_{\text{abs(up)}}(\nu, u)$ over the height of the column,

$$\mathbf{F}_{\text{abs}}(\nu) = \mathbf{B}_{s,\nu}(\nu, T_s) \mathbf{S}_\nu \int n_i(\mathbf{u}) d\mathbf{u}.$$

For a well-mixed gas, the number concentration of absorbing molecules of species X_i , $n_i(\mathbf{u}) = x_i n_{\text{air}}(\mathbf{u})$, where x_i is the mixing ratio and n_{air} is the number concentration of air molecules, so

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where $\int n_{\text{air}}(\mathbf{u}) d\mathbf{u}$ is the number of molecules of air per area in the atmospheric column.

The emission is assumed to be equal to the absorption of upwelling radiant energy reduced by the Boltzmann factor, $\exp[-h\nu(T(u)^{-1} - T_s^{-1})/k]$, where $T(u)$ is the local atmospheric temperature, a function of altitude. At a given altitude u in the atmosphere the volumetric rate of emission is

$$\mathbf{P}_{\text{emit(down)}}(\nu, \mathbf{u}) = \mathbf{P}_{\text{emit(up)}}(\nu, \mathbf{u}) = \mathbf{B}_{s,\nu}(\nu, T_s) x_i \mathbf{S}_\nu(\nu, \mathbf{u}) n_{\text{air}}(\mathbf{u}) \exp[-h\nu(T(u)^{-1} - T_s^{-1})/k]/2,$$

where the Boltzmann factor governing the emitted power adjusts for the local atmospheric temperature¹.

Then the local volumetric heating rate is the local difference between absorption and emission

$$\mathbf{P}_{\text{net}}(\nu, \mathbf{u}) = \mathbf{P}_{\text{abs(up)}}(\nu, \mathbf{u}) - \mathbf{P}_{\text{emit(down)}}(\nu, \mathbf{u}) - \mathbf{P}_{\text{emit(up)}}(\nu, \mathbf{u}),$$

and the cumulative change in upwelling irradiance from the surface to altitude u due to a specific absorption band is obtained from the vertical integral of the difference in absorbed and upward emitted power,

$$F_{\text{net (up)}}(\nu, u) = \int_0^u [P_{\text{abs(up)}}(\nu, u') - 2P_{\text{emit(up)}}(\nu, u')] du', \text{ so}$$

$$\mathbf{F}_{\text{net (up)}}(\nu, \mathbf{u}) = \mathbf{B}_{s,\nu}(\nu, T_s) \mathbf{S}_\nu(\nu, \mathbf{u}) x_i \int_0^u n_{\text{air}}(u') [1 - \exp[-h\nu(T(u')^{-1} - T_s^{-1})/k]] d\mathbf{u}',$$

and the net top of atmosphere forcing is given by the integral over the entire height of the vertical column.

1. Schwartz (2018) assumes that there is no contribution from downwelling irradiance, but the doubling up of the emission results from considering the change in the net flux arising from absorption and upward and downward emissions at layer du and then integrating from the surface to altitude u rather than calculating the contributions to the change in the net upward flux at du from sum of the integral of absorption and upward emission by the atmosphere from the surface to altitude u and the integral of downward emission from altitude u to the top of the atmosphere, then integrating over the whole column. Consequently it also omits the contribution to the net forcing from downward emission from above du .

Mlynczak et al. (2016) and Schwartz (2018) calculate radiative forcing at the tropopause by assuming that the absorption of blackbody terrestrial radiation by greenhouse gases is determined by the surface temperature at which the radiation was originally emitted and the spectral properties of greenhouse gases, using a radiative transmittance function based on the line strength and line shape of the absorption lines and the vertical optical mass, whilst, under conditions of local thermodynamic equilibrium arising from predominately collisional deactivation of excited greenhouse gas molecules up to 60 km altitude, the emission of radiation

at each layer of the atmosphere is given by the Planck blackbody function at the local atmospheric temperature, which decreases with height in the troposphere. Radiative forcing is given by the net change in radiative flux at the troposphere due to an increase in greenhouse gases, which is seen to create a transfer of radiative energy to the atmosphere and radiative imbalance at the tropopause. A linear relationship between the change in surface temperature in °C and radiative forcing is assumed.

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It is less easy to relate the net change in radiative flux at the troposphere due to an increase in greenhouse gases to a change in the average daily surface temperature. If it is assumed that the upwelling flux at the tropopause is equal to the flux at the top of the atmosphere, and the change in radiative flux at the tropopause due to the increase in greenhouse gases creates a proportionate change in terrestrial radiation absorbed by the atmosphere, or a proportionate change in the flux emitted by the Earth's surface, the estimated radiative forcing of 2.83 W/m² due to the increase in greenhouse gases from 1750 to 2011 (Myhre et al., 2013) would result in an increase in the flux emitted by the surface of between 4.3 and 4.9 W/m² and, applying the Stephan-Boltzmann Law, an increase in surface temperature of between 0.8 and 1.0 °C. This represents a climate sensitivity of around 0.32 °C/(W/m²), about one third of the climate sensitivity of 1.0 °C/(W/m²) used by IPCC 2013 (Flato et al., 2013) that was obtained from the mean regression-based values of 30 climate models, and 30% less than the climate sensitivity of 0.43 °C/(W/m²) used by Plass (1956) and Kaplan (1960).

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