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RESEARCH ARTICLE

A Dual-Loop Control Scheme for Energy-Efficient LED Lighting Systems

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Abstract

The escalating global population and depletion of energy resources have intensified the challenge of energy conservation, with buildings alone responsible for 30% to 40% of total energy consumption. Lighting, constituting up to 20% of global energy usage, necessitates innovative strategies for conservation. LED technology, renowned for its efficiency, serves as a pivotal solution. Various control methods, including proportional-derivative plus integral (PD+I) control, have been explored to enhance energy savings while ensuring visual comfort. Mathematical models, both linear and nonlinear, are employed for lamp characterization and control scheme design. Simulation results demonstrate the effectiveness of the proposed PD+I control in achieving energy efficiency across different scenarios.

KEYWORDS

Modelling, Simulation, Control, Illumination, Energy saving

1 | INTRODUCTION

The swift expansion of the global population, coupled with the exhaustion of energy resources and the consequential environmental repercussions, has elevated energy conservation to a formidable challenge. A substantial segment of worldwide energy utilization is attributed to the energy consumption in both residential and commercial structures. As indicated by data released by the United Nations Environment Programme (UNEP), buildings account for a substantial 30% to 40% of the world's total energy consumption[?]. A substantial contributor to global energy consumption is lighting, accounting for as much as 20% of the total energy consumed worldwide[?], with a noteworthy proportion emanating from both commercial and residential buildings. Consequently, there exists a pressing necessity to devise innovative strategies for energy conservation in lighting, integrating cutting-edge technologies and advanced control strategies.

Research has substantiated that the absence or surplus of lighting can impair visual acuity, modify contrast sensitivity, and influence ocular functions. Inadequate lighting fosters uninspiring and dim visual environments, while an excess of light can result in glare. Effective lighting not only diminishes energy consumption costs but also ensures visual comfort, empowering employees to perform their tasks without the undue strain caused by inadequate lighting in their surroundings. Iason et al. claim that working in conducive conditions, coupled with a sense of well-being, significantly boosts productivity[?].

LED technology stands out as a compelling light source owing to its prolonged lifespan, elevated energy and luminous efficiency, and the cost-effectiveness inherent in designing a lighting system with dimming capabilities.

Numerous studies have demonstrated the potential to reduce lighting energy consumption by implementing an indoor lighting control system with dimming capabilities. Petrov investigated the dependence of the change in the luminous flux of an LED lighting system on the electricity consumed where he presented a non-linear mathematical model[?]. While Le et al. present a mathematical model where the change in luminous flux depends on the electric current, but incorporates a light dependent resistance (LDR) sensor, which has the disadvantage of a slow response[?]. Researchers derived a third-order linear dynamic mathematical model for a 2-zone room fluorescent lamp lighting system[?]. They employed two main control approaches: computer-aided design and self-tuning techniques. Meanwhile, Copot utilized particle swarm optimization to fine-tune the PID controller for a lighting control system based on the model for a 2-zone room[?].

Researchers have proposed various control strategies to enhance energy savings. One such methodology involves the use of an intelligent lighting control system. Most of these strategies treat the lighting control problem as a constrained minimization problem and typically solve it using linear programming[?????].

The studies referenced in the preceding section do not address the LED lamp model or provide insights into the mapping of the relationship between illumination and dimming signals (voltage). In this study, we put forth a lighting control scheme designed for a dual-loop LED lamp. This scheme incorporates proportional-derivative control within the inner loop and integral control within the outer loop, all grounded in the lamp's mathematical model. Furthermore, we introduce a tuning strategy for the controller gains based on the design parameters of the standard second-order system. Notably, our assumption is that the lamp operates in isolation from any external lighting stimuli, whether natural or artificial. Thus, our approach is exclusively centered on the precise control of the lamp's lighting.

The structure of this research unfolds as follows: Section II we introduce some mathematical preliminaries, while Section III delves into the presentation of the mathematical model of the LED lamp as a static system. Moving forward to Section IV, we introduce a more comprehensive proportional-derivative-integral control scheme along with the accompanying tuning rule for the controller. Section V unveils the numerical results and simulations of the proposed scheme. The research concludes with a summarization of findings in the concluding section.

2 | PRELIMINARIES

2.1 | Linearization based on Taylor series

The act of replacing a non-linear system with its linear approximation is referred to as linearization[?]. The motivation for linearization stems from the dynamic behavior of many non-linear systems, where, within a range of variables, they can be approximated to linear system models.

Let's consider the case of a non-linear system with a state variable x and an output variable y, related by

$$y = h(x), \tag{1}$$

where the function $h : R \to R$ is continuous and differentiable; that is, $h \in C$. Let's consider x_0 as the operating point. If we expand h in the Taylor series around the point x_0, we obtain

$$y = h\left(x_0\right) + \frac{dh(x_0)}{dx}\left(x - x_0\right) + \text{higher-order terms.} \tag{2}$$

The linearization of $h(x)$ around the point x_0 involves replacing $h(x)$ with a linear approximation of the form

$$\hat{y} = \frac{dh(x_0)}{dx}\hat{x}, \tag{3}$$

where $y_0 = h(x_0)$, $\hat{y} = y - y_0$ and $\hat{x} = x - x_0$. Over a small range of $\hat{x}$, it is a good approximation of the curve $y = h(x)$ in the vicinity of the operating point x_0, see Fig. 1.

2.2 | Static system vs dynamic system

Before establishing the mathematical model that describes the behavior of the LED lamp, some concepts are introduced.

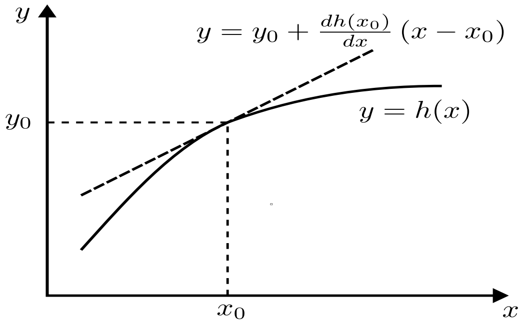

FIGURE 1 Graphical description of linearization.

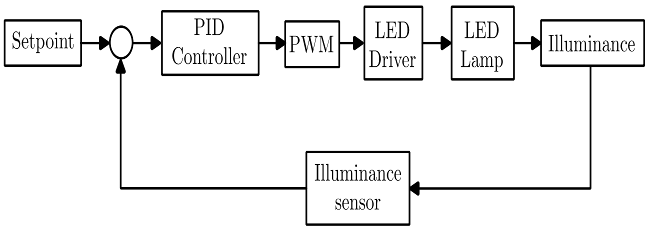

FIGURE 2 Lighting automatic control schematic.

A system whose response or output is due to present input alone is known as *static system*. The static system is also called the memoryless system. For a static or memoryless system, the output of the system at any instant of time depends only on the input applied at that instant of time, but not on the past or future values of the input. An example of a static system is the voltage-current relationship in a resistor (Ohm's Law), (4) describes the electrical behavior of resistor

$$v(t) = Ri(t). \tag{4}$$

The equation (4) states that the voltage value $v(t)$ at time t depends on the current at that same moment, multiplied by the resistance value. Unlike a dynamic system, the current output value only depends on the current state and not on previous states.

A system whose response or output depends upon the past or future inputs in addition to the present input is called the dynamic system. The dynamic systems are also known as memory systems. Any continuous-time dynamic system can be described by a differential equation. An electric circuit containing inductors and (or) capacitors is an example of dynamic system.

3 | MATHEMATICAL MODEL

3.1 | Dimmable LED lamp

A commercial dimmable LED lamp with the following features was chosen: rated voltage of 100-127 V, power consumption of 10 W, current consumption of 0.08-0.1 A, and luminous flux of 850 lm. This lamp has been chosen from a variety in the market due to its ability to adjust its brightness level.

A typical lighting automatic control system is shown in Fig. 2. The lighting environment considered in this system is consisted of a LED lamp, a working plane. A LED lamp is supplied from a LED driver, which is controlled by a microcontroller through PWM. Illuminance measured by an illuminance sensor (BH1750 sensor).

The lighting of a lamp in a natural environment is always exposed to external stimuli, whether natural or artificial. In other words, the illumination on a surface is the sum of the lamp's illumination plus external light, which can be from the sun (natural)

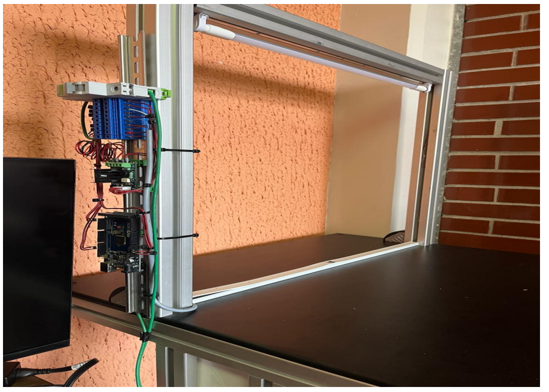

FIGURE 3 LED lamp experimental platform.

or from another lamp in its vicinity (artificial). From automatic control point of view, natural light is load disturbance, for this study, it is assumed that the lamp is isolated from any external source, whether natural or artificial.

3.2 | Conditions for measuring lamp illumination

In order to create a model for a lighting lamp, we systematically adjusted the supply voltage to the lamp dimmer in incremental steps. The ensuing response, measured using the BH1750 light sensor, was recorded utilizing a MEGA2560 data acquisition board. Illumination measurements were taken under the following conditions:

- The dimmable lamp shown in Fig. 3 was completely isolated from external illumination (natural or artificial).
- The Arduino MEGA2560 development board was used for acquiring signals from the light sensor.
- The BH1750 ambient light sensor with I2C bus interface and a wide range of resolution (1-65535 lux) was chosen for illumination measurements.
- In order to validate the measurements taken by the BH1750 sensor, the Kyoritsu Digital light meter model 5202 was used.
- The distance between the BH1750 ambient light sensor and the dimmable lamp was 60 cm in a direct vertical path to the center of the lamp.
- Illumination measurements were taken applying a voltage between 30.4 and 122.9 V to the lamp.

The measurements taken with the BH1750 sensor are shown in Fig 4 (dashed blue line). In order to validate the results obtained with the BH1750 sensor, illumination measurements were taken under the same conditions described earlier using a calibrated device, and the results are shown in Fig. 4 (red circles). The data collected from the measurements can be observed in table 1.

TABLE 1 Measurements carried out with BH1750 sensor, luxmeter, and correction of measurements in the sensor..

	Measurements			Measurements correction	
Number	**Voltage (V)**	**Sensor BH1750 (Lux)**	**Luxometer (Lux)**	**Error (%)**	**Corrected value (Lux)**
1	30.4	1263	1198	5.42	1204.6
2	61.5	1210	1160	4.31	1154.0
3	85.1	1110	1066	4.12	1058.7
4	101.8	823	794	3.65	784.9
5	116.3	600	575	4.34	572.2
6	122.2	342	328	4.26	326.2
7	129.9	85	81	4.93	81.1

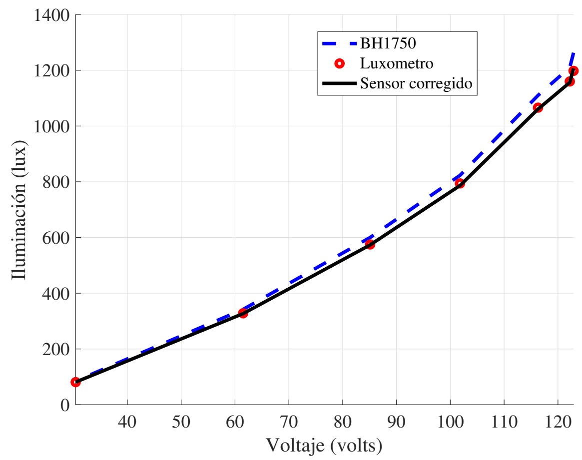

FIGURE 4 Error correction, using the BH1750 sensor and the luxometer.

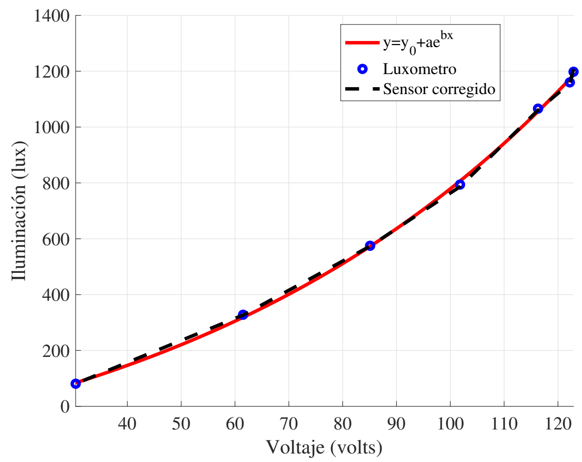

FIGURE 5 Mathematical model, corrected BH1750 sensor, and luxmeter.

As observed in table 1, there is a deviation in the measurement between the calibrated instrument (Kyoritsu Digital Light Meter Model 5202 luxometer) and the BH1750 sensor. The fifth column of table 1 shows the percentage error between both measurements (third and fourth columns). From table 1, an average error percentage of 4.63% was determined. To enhance the measurement accuracy with the BH1750 sensor, a correction was applied within the programming code when acquiring signals. By correcting with a 4.63% error, the values in the last column of table 1 were obtained.

The average error percentage based on the corrected value is 2%, achieving an average reduction of 2.63% through the programming code. To visualize the presented information, the values from the BH1750 sensor, the luxometer, and the corrected values with a 2% average error are plotted (see Fig. 4).

From Fig. 4, it is easy to observe that the correction applied to the BH1750 sensor measurement is appropriate, as mentioned, it can be carried out within the programming for acquiring measurements and implementing control strategies. Some control tuning techniques require knowledge of the mathematical model of the plant, in the next sections we will review the nonlinear and linear mathematical model for the lamp.

3.3 | Nonlinear modeling of LED lamp

By characterizing the behavior of the lamp using Sigma Plot software, the mathematical model that fits the data presented in sixth column from table 1 is

$$y = y_0 + y_1 e^{kx}, \tag{5}$$

where y_0 = –379.1, y_1 = 310.6 and k = 0.01316. In Figure 5, the comparison of the model (5) in dashed black line with the experimental results from the luxometer and the corrected sensor values is observed.

It is easy to observe in Fig. 5 that the model provided by Sigma Plot fits perfectly with the behavior of the lamp previously characterized through the conducted measurements.

3.4 | Linearization of nonlinear modeling of LED lamp

As seen in (5), the generated mathematical model is nonlinear. Assuming that the illumination in the dimmable LED lamp does not change concerning a specific operating point, a linear mathematical model can be used.

The Proposition 1 establishes the linear mathematical model characterizing the behavior of the LED lamp.

Proposition 1. *Consider the non-linear system*

$$y = y_0 + y_1 e^{kx},$$

assuming it is at least once differentiable at the operating point x_0, the linear mathematical model is

$$\tilde{y} = ax, \tag{6}$$

where $a = y_1 k e^{kx_0}$, $b = y_0 + y_1 e^{kx_0}(1 - kx_0)$ and $\tilde{y} = y - b$.

Proof. Consider the non-linear equation

$$y = y_0 + y_1 e^{kx},$$

by determining the Taylor series at the operating point x_0, considering only the terms of the first order, that is

$$y = h(x_0) + \frac{dh(x_0)}{dx}(x - x_0). \tag{7}$$

It is evident that

$$h(x_0) = y_0 + y_1 e^{kx_0},$$

and

$$\frac{dh(x_0)}{dx} = y_1 k e^{kx_0},$$

So that, by substituting into (7), the linear model is obtained

$$y = y_0 + y_1 e^{kx_0} + y_1 k e^{kx_0}(x - x_0),$$

defining $a = y_1 k e^{bx_0}$, $b = y_0 + y_1 e^{kx_0}(1 - kx_0)$ and $\tilde{y} = y - b$, resulting in (6). □

4 | PROPORTIONAL DERIVATIVE PLUS INTEGRAL CONTROL SCHEME

Consider control scheme shown in Fig. 6 for a LED lamp. The presented control strategy features a dual-loop control, where the internal or primary control loop is a proportional-derivative (PD) control, and the external or secondary control loop is an integral (I) control. It is assumed that the controller gains k_p, k_d, and k_I are positive.

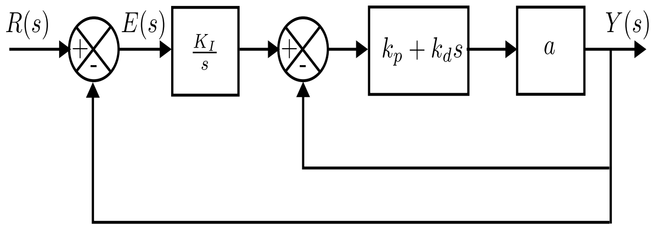

FIGURE 6 Proportional-derivative and integral control scheme.

4.1 | Tuning of the Proportional-Derivative Plus Integral Controller

The following proposition set the rules for tuning the proportional, integral and derivative gains of the controller.

Proposition 2. *Consider the static system described by equation*

$$\tilde{y} = a\,x,$$

with a dual-loop control scheme, proportional-derivative in the inner loop and integral in the outer loop. It is assumed that the control gains k_p, k_d, and k_I are positive. If ω_n and ζ satisfy the conditions, $\begin{bmatrix}\omega_n & \zeta\end{bmatrix} > 0$ and

$$\begin{bmatrix}\omega_n & \zeta\end{bmatrix}\begin{bmatrix}\lambda^2 & -\lambda \\ -\lambda & 0\end{bmatrix}\begin{bmatrix}\omega_n \\ \zeta\end{bmatrix} < -1, \tag{8}$$

for $\lambda > 0$, the control gains are given by

$$\begin{aligned} k_p &= \frac{1}{a(2\zeta\omega_n\lambda - \lambda^2\omega_n^2 - 1)}, \\ k_d &= \frac{\lambda}{a(2\zeta\omega_n\lambda - \lambda^2\omega_n^2 - 1)}, \\ k_I &= \lambda\omega_n^2. \end{aligned} \tag{9}$$

Proof. The closed-loop transfer function from control scheme in Fig. 6 is given as

$$\frac{Y(s)}{R(s)} = \frac{k_D k_I s + k_P k_I}{k_D s^2 + (1 + k_P + k_D k_I)s + k_P k_I},$$

where $k_P = ak_p$ and $k_D = ak_d$. The characteristic polynomial of this transfer function is denoted as

$$p(s) = s^2 + \frac{1 + k_P + k_D k_I}{k_D}s + \frac{k_P k_I}{k_D}. \tag{10}$$

The behavior and stability of a linear system are determined by the location of the roots of its characteristic polynomial in the complex-s plane. The characteristic polynomial for a standard second-order system is given by

$$q(s) = s^2 + 2\zeta\omega_n s + \omega_n^2. \tag{11}$$

By equating the characteristic polynomial (10) with the standard second-order system (11), we have

$$s^2 + \frac{1 + k_P + k_D k_I}{k_D}s + \frac{k_P k_I}{k_D} = s^2 + 2\zeta\omega_n s + \omega_n^2.$$

It is established that the derivative gain $k_D = \lambda k_P$. By equating terms and algebraic manipulation, we arrive at the equation

$$1 + k_P + \lambda k_P k_I = 2\lambda\zeta\omega_n k_P, \tag{12}$$

and the expression for the integral gain

$$k_I = \lambda\omega_n^2. \tag{13}$$

Substituting k_I from (13) into (12) and further algebraic manipulation result in the following expression for the proportional gain k_P,

$$k_P = \frac{1}{2\zeta\omega_n\lambda - \lambda^2\omega_n^2 - 1}. \tag{14}$$

Assuming positive controller gains, the denominator of (14) must satisfy the condition

$$2\zeta\omega_n\lambda - \lambda^2\omega_n^2 - 1 > 0, \tag{15}$$

that can be expressed in matrix form as follows

$$\begin{bmatrix}\omega_n & \zeta\end{bmatrix}\begin{bmatrix}\lambda^2 & -\lambda \\ -\lambda & 0\end{bmatrix}\begin{bmatrix}\omega_n \\ \zeta\end{bmatrix} < -1. \tag{16}$$

assuming that $\begin{bmatrix}\omega_n & \zeta\end{bmatrix} > 0$.

If the condition (16) is met, then the proportional gain k_P is given by (14). Previously, the derivative gain was defined as $k_D = \lambda k_P$, therefore

$$k_D = \frac{\lambda}{2\zeta\omega_n\lambda - \lambda^2\omega_n^2 - 1}$$

while the integral gain is given as $k_I = \lambda\omega_n^2$. Remembering that $k_P = a\, k_p$ and the derivative gain as $k_D = \lambda k_P$ and $k_D = a\, k_d$, thus, the gains are

$$\begin{aligned} k_p &= \frac{1}{a\left(2\zeta\omega_n\lambda - \lambda^2\omega_n^2 - 1\right)} \\ k_d &= \frac{\lambda}{a\left(2\zeta\omega_n\lambda - \lambda^2\omega_n^2 - 1\right)} \\ k_I &= \lambda\omega_n^2 \end{aligned}$$

□

4.2 | Stability of scheme control

Consider the characteristic polynomial of a standard second-order system (11). The roots of this characteristic polynomial are defined by

$$p_{1,2} = -\sigma \pm j\omega_d \tag{17}$$

with $\sigma = \zeta\omega_n$ and $\omega_d = \omega_n\sqrt{1-\zeta^2}$. In control theory, it is asserted that the standard second-order system exhibits stability if and only if its roots possess a negative real part, specifically, when $\sigma > 0$ in (17).

Proposition 2, delineating the tuning process for the PD+I control scheme, relies on the identification of roots in the complex plane by equating the characteristic polynomial (10) with that of a standard second-order system.

Then, from (9) for the proportional control gain, and after some algebraic manipulation, it becomes evident that

$$\sigma = \frac{1 + ak_p\lambda^2\omega_n^2 + ak_p}{2ak_p\lambda} \tag{18}$$

subject to the following established conditions: the control gains k_p, k_d, and k_I are positive, the value of λ must exceed zero, and $\begin{bmatrix}\omega_n & \zeta\end{bmatrix} > 0$. Additionally, it is important to highlight that a represents a characteristic parameter of the plant and is inherently positive. Consequently, the stability of the system is assured as long as Proposition 2 is satisfied.

TABLE 2 Values of gains for tuning conditions.

No	λ	ω_n	ζ	k_p	k_i	k_d
1	0.1	2.988	3.132	0.142	0.014	0.893
2	0.5	1.203	1.325	0.480	0.240	0.724
3	0.9	1.084	1.108	0.529	0.476	1.058

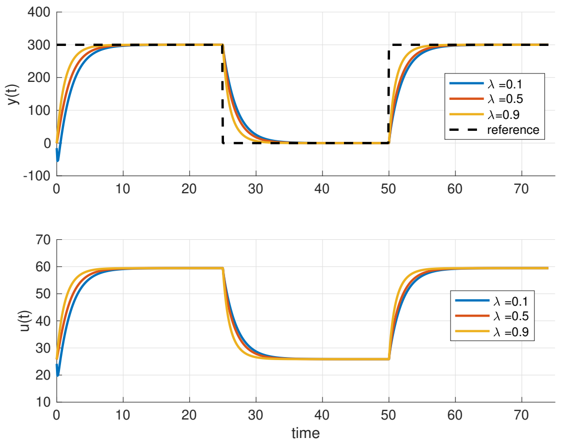

FIGURE 7 Simulation for different values of λ for a 300 lux input.

5 | NUMERICAL RESULTS AND SIMULATION

Consider the nonlinear system, as expressed in (5), where

$$y = y_0 + y_1 e^{kx}.$$

The operating point can be determined using the inverse function of the preceding equation

$$x = \frac{1}{k} \ln \left(\frac{y - y_0}{y_1} \right), \tag{19}$$

For a lighting output of $y = 300$ lux, and with the values $y_0 = -379.1$, $y_1 = 310.6$, and $k = 0.01316$, the corresponding operating point is $x_0 = 59.442$ V. By applying Proposition 1 to the operating point $x_0 = 59.442$ V, we obtain the model

$$\tilde{y} = 8.937x, \tag{20}$$

where $\tilde{y} = y + 231.234$.

We determine the control gains k_p, k_d, and k_I by utilizing (9), obtaining values for ω_n and ζ previously from condition (8) of Proposition 2 through Matlab for various values of λ. The resulting values are presented in Table 2.

The Fig.7 illustrates the simulation behavior of the proposed control scheme for a unit step input conducted in Matlab/Simulink. It can be observed that the control objective is achieved for different values of a, it is evident that the response with the best performance is for a value of $\lambda = 0.9$, while the worst case is for $\lambda = 0.1$.

In table 3, we can observe the settling time for each response, using the 2% criterion, meaning the time it takes for the response to reach 98% of its final value. Highlighting that a value of $\lambda = 0.9$ exhibits the best response time, reaching 98% of its final value in 7.03 seconds, making it the fastest. In contrast, for $\lambda = 0.1$, it takes 10.79 seconds to achieve the same value, indicating the slowest response, as depicted in Figure 8.

TABLE 3 Settling time for different values of λ.

No	λ	t_s (seg)
1	0.1	10.79
2	0.5	9.29
3	0.9	7.03

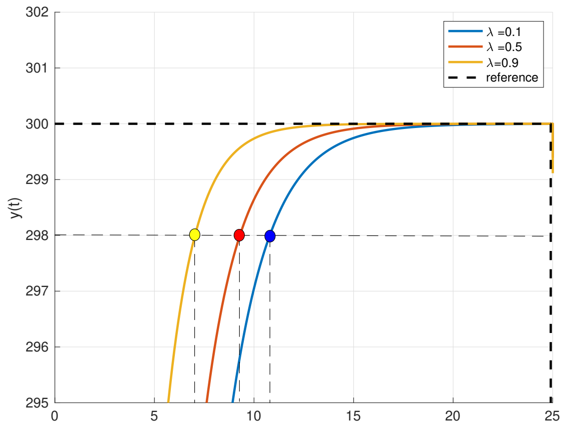

FIGURE 8 Settling times for different values of λ for an input of 300 lux.

6 | CONCLUSION

The developed work is based on the mathematical model of the isolated LED lamp from external light (natural or artificial). The obtained results lead to the conclusion that the dual-loop control scheme, with proportional-derivative in the inner loop and integral in the outer loop, along with the tuning rule for controller gains, enables a good system performance. As observed, the best response is achieved for small values of a, indicating a small derivative gain compared to the proportional gain. It is noteworthy that the response does not exhibit overdamping and has a smooth behavior, implying no glare generation. .

CONFLICT OF INTEREST

This work does not have any conflicts of interest.